Self-similarity in rain forests: Evidence for a critical state

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A cellular automata model of forest growth (the forest game) is shown to be able to reproduce the whole set of data available from a real rain forest where multifractality is present. It is conjectured that these fractal properties would be related with a self-organized critical state.

Fractal structures are ubiquitous in nature. Such structures exhibit self-similarity over a wide range of spatial and temporal scales, and their ubiquity suggests that the mechanisms implicit in their generation are robust and independent of the detailed physics involved. This observation led Bak, Tang, and Wisenfeld [1] to propose the theory of self-organized criticality (SOC). They explain that critical states are behind these spatiotemporal patterns and that such critical points are naturally reached with no external tuning of parameters. These states have been identified in physical systems [2] and in some complex biosystems as well as in evolutionary processes [3–5], but in the latter, experimental evidence is often difficult to obtain.

In order to test the theoretical predictions of SOC theory, we need to (1) identify in natural systems those properties characteristic of critical states and (2) construct simple (minimal) models able to reproduce such structures. In this paper both points are analyzed in relation to one of the most complex biosystems known, a rain forest.

In Fig. 1(a) a 50-ha (1 a = 10^2 m^2, 1 ha = 10^4 m^2) digitized map of Barro Colorado Island (BCI) rain forest is shown [6]. We have a 200×100 grid where $N_p = 2582$ low canopy survey points (LCP's) are plotted as black dots. They indicate that the height of the canopy was ≤ 10 m (the typical height is ≈ 50 m). These low canopy points are linked with the recent formation of a gap, due to treefall. (Figure 2, discussed later, shows a plot of gap sizes.) Such dynamical phenomena have been shown to

be very important in the maintenance of high diversity levels [7].

We first estimate the fractal dimension D_0 of our system Ω (the BCI forest). If $M(r)$ is the number of LCP's in an $L \times L$ square, we expect a scaling behavior in density as $\rho = M(L)/L^2 \approx L^{-D_0}$. In BCI, $D_0 = 1.85$ for $L < L_c \approx 60$ m, and for $L \gg L_c$ a trivial scaling with a slope equal to 2 is obtained. Using box counting Ω was partitioned into $l \times l$ boxes ($1 \leq l \leq 20$ pixel units), and two different scales were observed: $S_1 = \{5 < r < 30$ m$\}$ and $S_2 = \{30 < r < 60$ m$\}$. For S_2 we have slopes $d_{02} = 1.86 \pm 0.05$ and $d_{01} = 1.46 \pm 0.06$ for S_1. A continuous drift is observed between both linear scalings. This result is confirmed by means of the correlation function $C(r)$, defined as

$$C(r) = \frac{1}{N} \sum_{\mathbf{r}'} \rho(\mathbf{r}) \rho(\mathbf{r} + \mathbf{r}') , \qquad (1)$$

(here $r = |\mathbf{r} - \mathbf{r}'|$) where $\rho(\mathbf{r}) = 1$ for a black site, and 0 otherwise. For a two-dimensional fractal object, we have $C(r) \approx r^{-\nu}$ where $\nu = 2 - D_0$ and in our system we have $\nu_1 \approx 0.68 \pm 0.02$ for $r \in S_1$ and $\nu_2 \approx 0.21 \pm 0.04$ for $r \in S_2$ as expected. Two characteristic length scales are then observed [8] [Fig. 3(a)].

Now in order to characterize multifractality [9], a measure μ needs to be defined on our system Ω. The total size of Ω is partitioned into a set $\{B_i\}$ of pieces of length l, and a probability measure p_i is assigned to every piece (i.e., $p_i = \int_{B_i} d\mu$). For our system, we take the fraction of
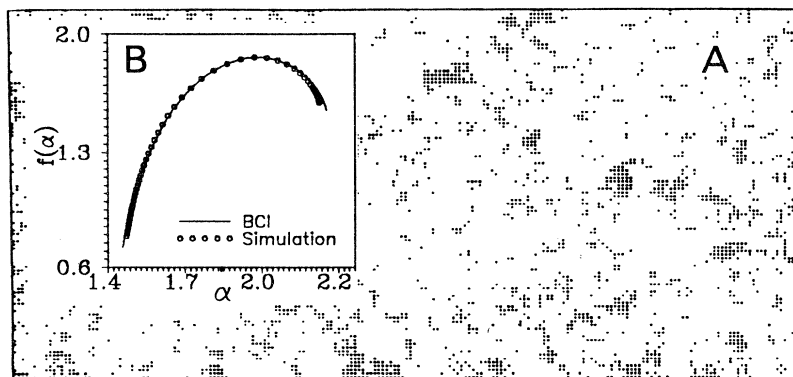

FIG. 1. (A) 1000×500 m snapshot of the BCI rain forest (Panama), showing LCP's as black dots; (B) $f(\alpha)$ spectrum for BCI and a 100×100 FG simulation; here $P_b = 0.5$, $P_d = 0.013$, and $\gamma = 2.5$.
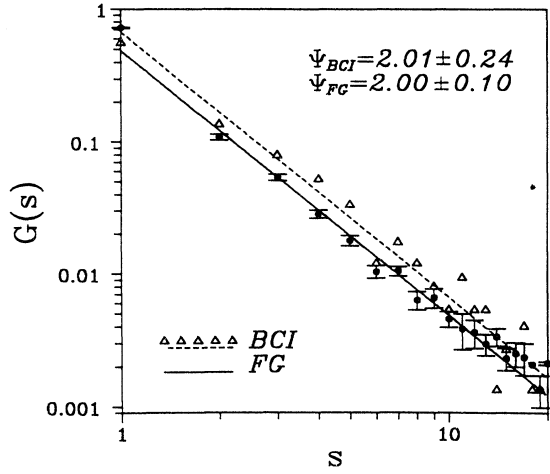
FIG. 2. Power-law distribution of gap sizes from BCI forest (dashed line) and FG (five 200×100 samples were used; parameters as in Fig. 1).

LCP's N_i inside B_i, i.e., $p_i = N_i / N_p$. If Ω is multifractal, it can be described as $\Omega = \cup_\alpha \Omega_\alpha$, where $\{\Omega_\alpha\}$ are subsets with scaling $p_i \approx l^\alpha$.

Following standard methods [9], the generalized dimensions can be estimated. Using the q-order moments $M_q(\Omega) = \sum_i P_i^q$, Ω will be a multifractal if the scaling $M_q(\Omega) \approx l^{-\tau(q)}$ holds. The Legendre transform $f(\alpha)$ of $\tau(q)$ gives us a universal characterization of our system [9], and is shown in Fig. 1(b). As usual, we have $\alpha(q) = \partial_q [(q-1)D_q]$ and

$$f(\alpha) = q(\alpha) - [q(\alpha) - 1]D_{q(\alpha)} . \tag{2}$$

The fractal patterns that have been found in BCI may be a fingerprint of a system in a critical state. In such a case, a power-law distribution of gap sizes $[G(s)]$ is expected to occur (like avalanche sizes in sandpiles). For the BCI forest, we found that $G(s) \approx s^{-\psi}$ with $\psi = 2.01 \pm 0.24$. (See Fig. 2.)

Our results strongly suggest the possibility that BCI might be a snapshot of an ecosystem poised at a critical state. In fact, the BCI study was mainly motivated by the problem of how rain forests are organized in space and time. The theoretical prediction [10] is that they are systems in a nonequilibrium state. The previous results seem to confirm that such a nonequilibrium (critical) state should be present. In order to see if SOC is involved as the mechanism underlying the fractal structure of BCI, a cellular automata (CA) model—the forest game—(FG) [11] was used.

Here a two-dimensional $L \times L$ lattice $\Lambda(L)$ is used, on which trees grow and compete for resources (periodic boundaries are used). The size of each tree is specified at a given time step t by $S_t(i,j)$, where $(i,j) \in \Lambda(L)$. As in other models [12] the value of $S_t(i,j)$ can be a real number instead of belonging to a discrete set of states. In our CA model, we replace the 186 different tree species known in BCI [6] by a single type of tree. Here each point is a 5×5 m square in the BCI plot. Four basic
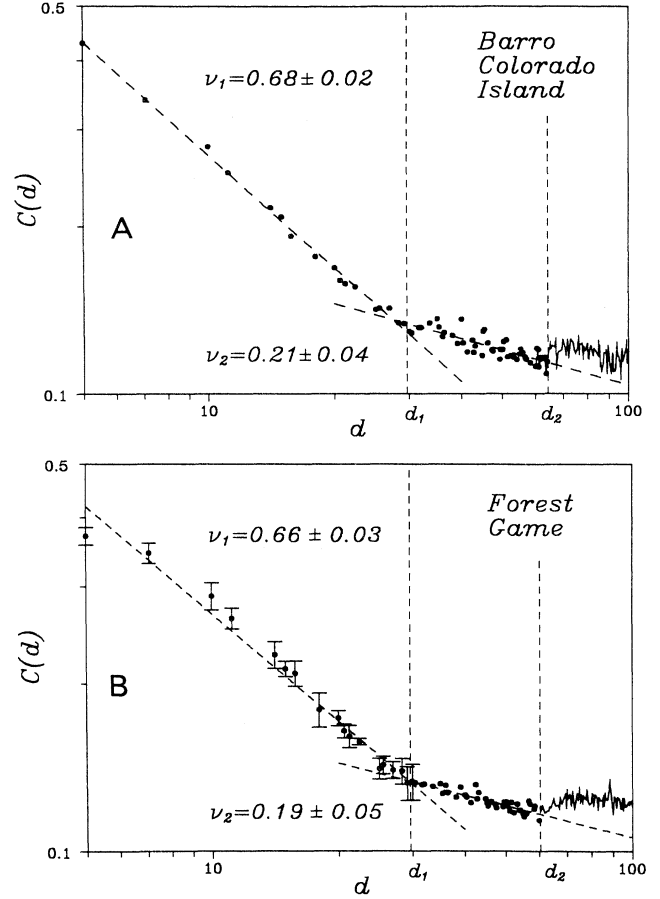

FIG. 3. (A) $C(r)$ for the BCI forest; (B) $C(r)$ for the FG automaton (parameters as in Fig. 1), averaged over ten 200×100 simulations.

rules are defined (and asynchronously applied). (a) *Birth:* a new tree can appear at any empty lattice site with probability P_b. The size of the newborn tree is the minimum one S_0. (Here $S_0 = 0.1$.) (b) *Death:* a tree will be removed according to a probability P_d but also if $S_t \geq S_c$, a maximum size. (c) *Growth:* tree sizes are updated according to

$$S_{t+1}(i,j) = S_t(i,j) + \Theta \left[1 - \frac{\gamma}{8} \sum_{\langle r,s \rangle} S_t(r,s) \right] \tag{3}$$

(here γ gives the interaction strength and $\langle r,s \rangle$ stands for the eight nearest neighbors). Here $\Theta(z) = z$ if $z > 0$, and $\Theta(z) = 0$ if $z < 0$: if the screening is too high, growing is not allowed. As we can see, the rule is linear except for the threshold. (d) *Gap formation:* when a tree dies, a gap in the canopy is formed. Not only is the dead tree removed, but also all the nearest neighbors $S_n(r,s)$ in a circle $B(R)$ of radius R such that $\sum_{B(R)} S_t(r,s) \leq S_t(i,j)$. The mass of neighbors removed is then proportional to the size of the falling tree, and at most equal to it. R is determined through the previous inequality. In our mod-
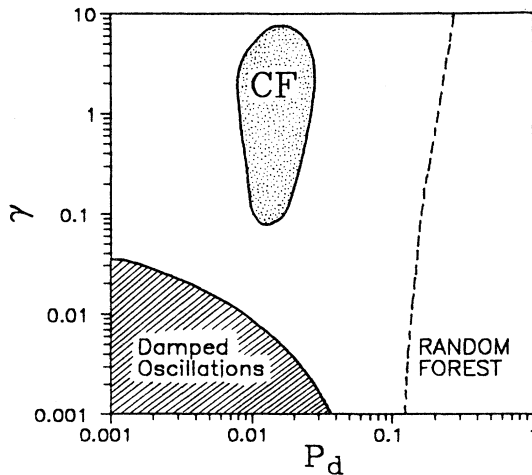
FIG. 4. Parameter space of the FG automaton, calculated from 100×100 lattices ($P_b = 0.5$). Here CF = complex forest.

el LCP's are those where $S_t(i,j) = 0$. Starting from a random initial condition [some trees of size S_0 are randomly scattered over $\Lambda(L)$], the system evolves to a steady state where all statistical quantities become stationary after $t > 75$–100 steps. We have used lattice sizes up to $L = 256$, but most calculations were performed using $L = 100$.

The phase space of our CA is shown in Fig. 4, for different (γ, P_d) values and [13] $P_b = 0.5$. For all these points the $f(\alpha)$ spectrum was calculated as well as $C(r)$, $G(s)$, and other quantities of interest such as the Fourier spectrum $S(f)$ of biomass fluctuations, i.e., of $B(\Omega, t) = \sum_{(i,j)} S_t(i,j)$ and the distribution of tree sizes $N(S)$ [14]. The following three main domains are observed.

(1) Damped oscillations. For $P_d, \gamma \ll 1$, screening is almost absent and the forest is highly synchronized, with strong damped fluctuations in biomass, small correlations and well defined peaks [15] in $S(f)$; the period of oscillation τ is shown to scale with S_c as $\tau \approx S_c^{-\kappa}$ with $\kappa \approx 1$.

(2) Random forest. For $P_d \to 1$, the forest dynamics

behaves as white noise and $D_0 = 2$. When $\gamma \to 0$, we observe a canonical distribution of tree sizes, i.e., $N(T) = \exp(-T/T^*)$ where $T^* = \langle T \rangle$. Hence $C(r)$ is constant, i.e., no correlations are observed.

(3) Complex forest. This domain is the most interesting one. A snapshot of the FG (at $t = 150$) for $Q^* = (P_d^* = 0.013,\ \gamma^* = 2.5)$ is shown in Fig. 5. In this domain *all* the observed properties of the BCI system (in particular for Q^*) are very well reproduced: (i) $C(r)$ shows the same two characteristic length scales, as shown in Fig. 3; here we have $\nu_1 \approx 0.66 \pm 0.03$ and $\nu_2 \approx 0.19 \pm 0.05$ for Q^* and $L_c \approx 60$ m; (ii) $G(s)$ also shows a power-law behavior with $\psi \approx 2.00 \pm 0.10$ (Fig. 2); (iii) tree sizes show a scaling $N(S) \approx S^{-\eta}$ with $\eta \approx 1.00 \pm 0.02$ (this value is not available for BCI), and (iv) the fractal dimension is $D_0 \approx 1.87 \pm 0.05$ with a very good agreement in $f(\alpha)$ as shown in Fig. 1(b). An additional quantity (not available from BCI data) is the temporal behavior of $B(\Omega, t)$. The Fourier spectrum shows $f^{-\beta}$ noise for a wide region of parameter space around the complex forest (CF) domain, but the largest β are obtained only in this domain. The CF region is formed by those systems closer to $1/f$ noise, more precisely those where $\beta > 0.9$. For Q^* we have $\beta = 1.00$.

The white area around the CF shows intermediate properties, in particular for $P_d \ll 1$ and high γ, the forest dynamics becomes "frozen," very short correlations are present, and anomalous (nondecreasing) distributions of $N(S)$ and $G(s)$ are obtained.

In summary, we have shown that the BCI forest is a multifractal and that a simple CA model can account for a very important piece of rain forest complexity, the spatial structure of forest gaps. A typical value of tree death rates obtained from field studies [15] gives $P_d \approx 0.01$–0.02, which belongs to the parameter domain where the complex forest is seen. A very broad interval of γ values is involved, suggesting that the CF should be the *generic* behavior of real rain forests, as expected in SOC theory.

As previous mentioned, gaps are known to be a source of diversity in rain forests [7,10]. For a long time, it has been conjectured that some kind of nonequilibrium dynamics is behind the enormous diversity of these ecosystems [10]. The FG model provides a quantitative frame-
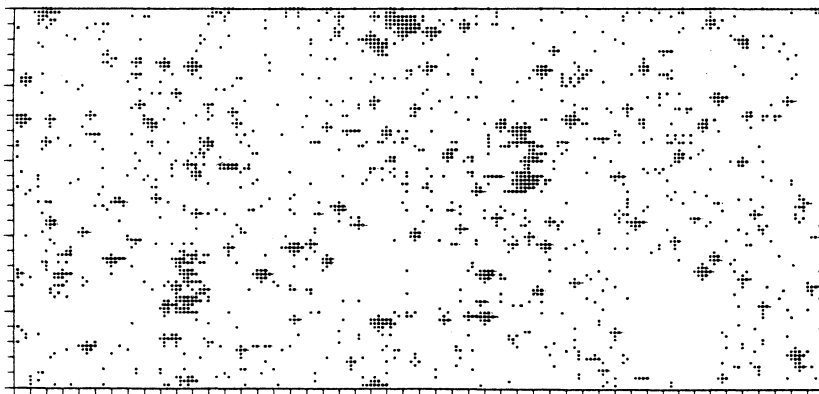

FIG. 5. A 200×100 snapshot of the FG automaton, for (P_d^*, γ^*), at $t = 150$.

work able to give a nontrivial relationship between diversity and critical states [16,17]. In this sense the BCI rain forest could be an example of a complex biosystem poised at *the edge of chaos.*

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[8] These correspond to the size of large gaps and to the characteristic distance between them. Gap distributions are known to be nonrandom (clustered) and nonrelated to external factors such as topography [see, e.g., D. Lieberman, M. Lieberman, R. Peralta, and G. Hartshorn, J. Ecol. **73**, 915 (1985)].

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[13] Here P_b is a measure of how fast the gap is filled (in the forest, until the canopy is higher than 10 m). Other intermediate values only act by slightly shifting the CF domain.

[14] Here a power-law behavior is observed in all nondisturbed rain forests, as well as in the FG automaton at the critical state.

[15] More precisely, this domain includes those (γ, P_d) values where at least 10 oscillations in $B(\Omega, t)$ are observed.

[16] Several alternative models have been analyzed, where different rules were used and equivalent results are obtained.

[17] Recent simulations using up to 50 different species with different γ values show that our results are robust: multifractality and scaling laws are typical. In these simulations high diversity is observed at the CF domain, in agreement with the previous results [Solé and Manrubia (unpublished)].